Digital adoption, population ageing, and age-specific structural change: A two-sector model with mobility frictions

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Abstract

This paper develops a tractable two-sector model to study how population ageing interacts with digital adoption to shape age-specific structural change. The labour force consists of young and old workers. Each sector combines the two age groups through a CES technology, while imperfect reallocation is captured by a logit sector-choice block that generates age-specific mobility wedges. Digital adoption affects older workers through two channels: it raises their sector-specific effective productivity and lowers their relative barrier to switching sectors. The model yields a closed-form "sorting" restriction that links a weighted difference in age-specific employment log-odds to primitives (age-augmenting technologies and mobility wedges). We derive sharp sign conditions under which digital adoption reduces age segmentation and under which ageing pushes older workers toward or away from a sector. A simple planner problem shows that optimal adoption rises with ageing when adoption removes more misallocation in older economies.

Keywords: Digital adoption; population ageing; labour mobility frictions; sectoral reallocation

JEL Classification: J14; J24; J62; O33; O41

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1. Introduction

Picture a familiar scene in many economies today. A firm has good demand for its product, but its workforce is older than it used to be. Some jobs are still done the same way as twenty years ago, but others now run through digital tools: online matching platforms, remote-work systems, cloud-based software, or automated equipment. Managers often say the same thing in two different sentences. First: “older workers are reliable and experienced.” Second: “it is hard to move people across tasks and departments, and training takes time.” That tension is not only a firm story. It is also a macro story. Population ageing changes who works, where they work, and how easily labour reallocates when the economy tilts from one sector to another.

This paper asks a narrow question with broad relevance: when the workforce becomes older, can digital adoption reduce age segmentation across sectors, or does ageing simply deepen it? We study this question in a deliberately small model that still captures two facts that often move together. The first fact is structural change: employment and output shares shift across sectors over long horizons (Kongsamut, Rebelo and Xie, 2001; Ngai and Pissarides, 2007; Duarte and Restuccia, 2010; Herrendorf, Rogerson and Valentinyi, 2014). The second fact is that adjustment is costly: labour does not flow freely to where wages are highest, and wage gaps can persist because switching sectors is painful and slow (Artuç, Chaudhuri and McLaren, 2010; Dix-Carneiro, 2014; Caliendo, Dvorkin and Parro, 2019). The novelty here is to place these two facts inside a simple framework where age matters, and where digital adoption can affect older workers through two separate channels: productivity and mobility.

Our starting point is the structural transformation literature. Classic contributions show that balanced growth is consistent with stable aggregates but misses large reallocations across broad sectors (Kongsamut, Rebelo and

Xie, 2001). Modern multi-sector models explain why sectoral employment shares can trend even in steady growth, driven by differential productivity growth, income effects, and relative price movements (Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008; Duarte and Restuccia, 2010; Herrendorf, Rogerson and Valentinyi, 2014; Buera and Kaboski, 2012). These papers typically treat labour as a single factor. That is often fine for long-run sectoral shares, but it is less informative when the key margin is who reallocates—for example, whether older workers follow the same paths as younger workers, or become trapped in shrinking sectors.

A second strand studies labour adjustment frictions. Structural empirical work shows that moving across sectors can entail large switching costs, generating slow transitions and nontrivial welfare dynamics (Artuç, Chaudhuri and McLaren, 2010). Related dynamic general equilibrium frameworks embed mobility frictions to quantify adjustment to shocks and policy changes (Dix-Carneiro, 2014; Caliendo, Dvorkin and Parro, 2019). Our contribution is not to re-estimate such models. Instead, we use the same basic logic—that sectoral mobility is costly—but we let these costs differ by age and respond to digital adoption.

A third strand connects demographics and technology. A growing empirical literature argues that ageing can change the direction and intensity of automation adoption (Acemoglu, Restrepo and Krueger, 2022). This line of work is important because it suggests that ageing does not only reduce labour supply; it can also shift technology choices. But it leaves open a closely related question: even if technology adapts, does it help older workers move across sectors and tasks, or does it mostly replace them? Our paper takes a small step toward this question by focusing on reallocation across two sectors and on age-specific mobility frictions.

Finally, there is a large body of work on digital technologies and labour-market outcomes. Information technology can raise productivity, especially when complemented by organisational change (Brynjolfsson and Hitt, 2000; Bloom, Sadun and Van Reenen, 2012). Digital connectivity can shift employment and task allocation in measurable ways (Akerman, Gaarder and Mogstad, 2015; Hjort and Poulsen, 2019). Remote-work feasibility, highlighted during the COVID period, differs sharply across occupations and industries (Dingel and Neiman, 2020). At the worker level, older cohorts can face different incentives and constraints in adopting new skills: computer use and training decisions are tied to retirement horizons and expected returns to re-skilling (Friedberg, 2003; Bartel and Sicherman, 1993). These studies motivate our modelling choice to let digital adoption affect older workers not only through productivity but also through mobility or switching barriers.

We build a static two-sector model with two age groups in the labour force, young and old. An ageing index λ captures the composition of the workforce. Digital adoption is summarised by an index $D \geq 1$ that is costly through a convex resource cost $\Phi(D)$. The key modelling choice is to allow digital adoption to influence older workers in two distinct ways.

First, digital adoption can raise the effective productivity of older workers in a sector-specific manner. This reflects the idea that some sectors are more "digitally compatible" for older workers—for example, because tasks can be reorganised, monitored, or supported by digital tools. Second, digital adoption can reduce the cost older workers face when switching sectors, for example by improving job matching, lowering search costs, enabling remote work, or improving the effectiveness of retraining programmes.

To keep the model transparent, we assume both mechanisms have simple reduced-form representations that feed into equilibrium allocations. On the production side, each sector uses young and old labour with a CES aggregator, so the within-sector age mix responds to relative wages and relative efficiency. On the mobility side, we microfound sectoral wedges with a logit sector-choice block: within each age group, workers have idiosyncratic sector tastes and face age-specific barriers to entering a sector. This yields closed-form odds ratios for sectoral employment shares. The combination delivers a tractable restriction linking (weighted) log-odds of sector-1 employment across age groups to primitives (technology and wedges).

The payoff is that we can state sharp, interpretable conditions for when digital adoption reduces age segmentation, when ageing pushes older workers toward or away from a sector, and when adoption becomes more valuable in older economies.

The paper makes three points. First, we derive an equilibrium "sorting" restriction that links a weighted difference in age-specific log-odds of sectoral employment to (i) relative age-augmenting technologies across sectors and (ii) age-specific mobility wedges. This creates a simple bridge between multi-sector structural change models (Ngai and Pissarides, 2007; Herrendorf, Rogerson and Valentinyi, 2014) and the adjustment-cost tradition (Artuç, Chaudhuri and McLaren, 2010; Dix-Carneiro, 2014) in an explicitly age-heterogeneous environment.

Second, digital adoption affects older workers through a productivity channel and a mobility channel. We provide an "if and only if" sign condition under which higher D increases the relative odds that older workers are employed in one sector compared with younger workers. The condition is simple: digital adoption reduces age segmentation if it sufficiently relaxes old-worker mobility barriers and/or raises old-worker efficiency in the relevant sector more than in the alternative sector. This speaks to the broader debate on whether technology complements or substitutes for older labour (Friedberg, 2003; Acemoglu, Restrepo and Krueger, 2022).

Third, ageing affects the equilibrium both by tightening old-worker mobility (switching becomes harder) and by eroding old-worker efficiency in a potentially sector-biased way. We show how these forces compete in a single inequality, yielding a transparent prediction for the direction of age-specific reallocation. We then connect the allocation results to an endogenous adoption choice in a simple planner problem. Under a mild cross-partial condition—interpretable as "adoption removes more misallocation when the workforce is older"—the optimal adoption level rises with ageing.

There are many reasons why older workers might be concentrated in some sectors: sector-specific human capital, rigid job ladders, health constraints, or discrimination. Our model does not try to include all of these margins. Instead, it isolates two mechanisms that can be stated in plain terms and that can be linked to policy: digital adoption can change how productive older workers are, and it can change how hard it is for them to move. These are the two mechanisms policymakers often have in mind when they talk about "digital inclusion" or "re-skilling" for ageing societies. The model provides a disciplined way to see when these policies can plausibly reduce segmentation and when they cannot.

Section 2 sets up the economy, defines technologies and mobility wedges, and characterises equilibrium. We then derive the age-specific sorting restriction and use it to obtain comparative statics for digital adoption and ageing. We also discuss a simple extension that generates complementarity between ageing and the marginal impact of adoption. Section 3 concludes with policy recommendations provided.

2. Model

2.1 Set up

Time is static. There are two age groups in the labour force: young (y) and old (o). Let total supplies be (L^y, L^o), taken as exogenous. Define an ageing index:

$$\lambda \equiv \frac{L^o}{L^y + L^o} \in (0,1). \tag{1}$$

An increase in λ represents an older workforce (holding $L^y + L^o$ fixed).

There is a representative household that owns firms and consumes the final good C. To keep focused on structural change and labour allocation, labour supplies are inelastic and utility is quasi-linear:

$$U = C - \Phi(D), \tag{2}$$

where $D \geq 1$ is a digital adoption (or re-skilling / matching) index and $\Phi(D)$ is the resource cost of adoption, paid in units of the final good. We assume Φ is increasing and convex:

Assumption 1 (Adoption cost) $\Phi'(D) > 0$ *and* $\Phi''(D) > 0$ *for* $D > 1$*, with* $\Phi(1) = 0$.

We endogenises D as the solution to a simple planner problem; elsewhere D can be read as a policy variable or an equilibrium object pinned down by technology and institutions.

There are two intermediate sectors $j \in \{1,2\}$ producing goods Y_1, Y_2 under perfect competition. A final-good producer aggregates these intermediates into the numeraire good Q via a CES technology:

$$Q = \left[\gamma_1^{\frac{1}{\varepsilon}} Y_1^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_2^{\frac{1}{\varepsilon}} Y_2^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \varepsilon > 0,\ \varepsilon \neq 1,\ \gamma_j > 0. \tag{3}$$

Let the final-good price be normalised to one and denote intermediate prices by (P_1, P_2). Cost minimisation yields the familiar relative demand condition:

$$\frac{P_1 Y_1}{P_2 Y_2} = \frac{\gamma_1}{\gamma_2}\left(\frac{P_1}{P_2}\right)^{1-\varepsilon}. \tag{4}$$

Each intermediate sector uses young and old labour with constant returns. Let (L_j^y, L_j^o) denote age-specific labour allocated to sector j. Output is:

$$Y_j = A_j \left[(\theta_j^y L_j^y)^{\frac{\eta-1}{\eta}} + (\theta_j^o(D,\lambda)\, L_j^o)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad \eta > 0,\ \eta \neq 1,\ A_j > 0,\ \theta_j^y > 0. \tag{5}$$

Parameter η is the elasticity of substitution between young and old labour within a sector.

We allow older-worker efficiency to depend both on ageing and on digital adoption:

$$\theta_j^o(D,\lambda) = \bar{\theta}_j^o \cdot \underbrace{(1-\lambda)^{\rho_j}}_{\text{“effective ageing”}} \cdot \underbrace{D^{\mu_j}}_{\text{digital complementarity}}, \qquad \bar{\theta}_j^o > 0,\ \rho_j \geq 0,\ \mu_j \geq 0. \tag{6}$$

The term $(1-\lambda)^{\rho_j}$ captures the idea that, as the workforce becomes older, the average health/physical capacity (or task suitability) of the marginal old worker declines in some sectors (larger ρ_j). The term D^{μ_j} captures sector-specific complementarity between digital tools and older workers. When μ_j is large, sector j benefits more from re-skilling, remote work, better matching, or automation that is relatively friendly to older workers.

Labour can be allocated across sectors but reallocation is not frictionless. We make the wage wedge interpretable, we microfound it using a simple sector-choice block.

For each age group $a \in \{y, o\}$, a unit mass of workers chooses between sectors $j \in \{1, 2\}$. A worker of age a who works in sector j earns wage w_j^a but pays a (possibly negative) utility cost $\tau_j^a(D, \lambda)$, measured in log wage units. In addition, workers draw i.i.d. idiosyncratic taste shocks ϵ_{ij}^a across sectors. Assume ϵ_{ij}^a follows a type-I extreme value distribution with scale parameter $\nu_a > 0$ (a standard device that yields logit shares).

The indirect utility of choosing sector j is:

$$v_{ij}^a = \ln w_j^a - \tau_j^a(D, \lambda) + \nu_a \, \epsilon_{ij}^a. \tag{7}$$

Let x^a denote the share of age-a labour working in sector 1:

$$x^a \equiv \frac{L_1^a}{L^a}, \qquad 1 - x^a = \frac{L_2^a}{L^a}. \tag{8}$$

The logit structure implies a closed-form expression for the odds ratio:

$$\ln\left(\frac{x^a}{1-x^a}\right) = \frac{1}{\nu_a}\left[\ln\left(\frac{w_1^a}{w_2^a}\right) - \Delta\tau^a(D,\lambda)\right], \qquad \Delta\tau^a(D,\lambda) \equiv \tau_1^a(D,\lambda) - \tau_2^a(D,\lambda). \tag{9}$$

Equivalently,

$$\frac{w_1^a}{w_2^a} = \exp\big(\Delta\tau^a(D,\lambda)\big)\left(\frac{x^a}{1-x^a}\right)^{\nu_a}. \tag{10}$$

Equation (10) indicates that when $\Delta\tau^a$ is large (sector 1 is harder to access, or sector 2 is easier), sector 1 must pay a higher wage to attract workers. The elasticity parameter ν_a governs how strongly employment shares respond to wage differences.

To capture the idea that digital tools reduce switching/search costs for older workers (online matching, remote work, re-training), assume

$$\Delta\tau^o(D,\lambda) = \overline{\Delta\tau}^o - \kappa \ln D + \chi\lambda, \qquad \kappa > 0, \; \chi \geq 0. \tag{11}$$

The term $-\kappa \ln D$ means higher adoption lowers the relative barrier into sector 1 for old workers. The term $\chi\lambda$ allows ageing itself to raise mobility frictions for older workers (e.g., switching becomes harder as the average old worker becomes older). For young workers, we keep a simpler reduced form:

$$\Delta\tau^y(D,\lambda) = \overline{\Delta\tau}^y, \tag{12}$$

though nothing prevents allowing digital adoption to matter for young workers as well.

Intermediate producers are perfectly competitive and choose inputs to minimise costs given (P_j, w^y_j, w^o_j). It is convenient to work with the dual of the CES aggregator in Equation (5). Define the sector-j effective labour composite

$$E_j \equiv \left[(\theta^y_j L^y_j)^{\frac{\eta-1}{\eta}} + (\theta^o_j(D,\lambda)\, L^o_j)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \qquad Y_j = A_j E_j. \tag{13}$$

Lemma 1 (Unit cost and conditional factor demands) *Fix a sector j and suppress the subscript j where no confusion arises. Given wages (w^y, w^o) and technology shifters $(\theta^y, \theta^o) \equiv (\theta^y, \theta^o(D,\lambda))$, define the CES "effective labour" composite*

$$E = \left[(\theta^y L^y)^{\frac{\eta-1}{\eta}} + (\theta^o L^o)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \qquad \eta > 0,\ \eta \neq 1. \tag{14}$$

Then: (i) the unit cost of one unit of E is

$$\omega(w^y, w^o; \theta^y, \theta^o) = \left[\left(\frac{w^y}{\theta^y}\right)^{1-\eta} + \left(\frac{w^o}{\theta^o}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{15}$$

(ii) if sectoral output is $Y = A\,E$ with $A > 0$, then the unit cost of Y is $c = \omega/A$ and, under perfect competition, $P = c$, i.e.

$$P = \frac{1}{A}\left[\left(\frac{w^y}{\theta^y}\right)^{1-\eta} + \left(\frac{w^o}{\theta^o}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{16}$$

(iii) conditional factor demands satisfy

$$\frac{L^o}{L^y} = \left(\frac{\theta^o}{\theta^y}\right)^{\eta-1}\left(\frac{w^y}{w^o}\right)^{\eta}. \tag{17}$$

Proof. *Step 1: Set up the cost-minimisation problem*: For any target level

$\bar{E} > 0$, consider the cost-minimisation problem:

$$\min_{L^y \geq 0,\, L^o \geq 0} \; w^y L^y + w^o L^o \quad \text{s.t.} \quad \left[(\theta^y L^y)^{\frac{\eta-1}{\eta}} + (\theta^o L^o)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \geq \bar{E}. \tag{18}$$

Since the aggregator is homogeneous of degree one in (L^y, L^o), the constraint binds at optimum. Let

$$\rho \equiv \frac{\eta - 1}{\eta} \in (-\infty, 1) \setminus \{0\}, \qquad \text{so that} \qquad \frac{\eta}{\eta - 1} = \frac{1}{\rho}.$$

Then the constraint can be written equivalently as:

$$(\theta^y L^y)^\rho + (\theta^o L^o)^\rho \geq \bar{E}^\rho. \tag{19}$$

Because ρ may be negative when $\eta \in (0, 1)$, it is useful to keep ρ explicitly; the Lagrange method works for all $\eta > 0, \eta \neq 1$.

Step 2: Lagrangian and first-order conditions: Form the Lagrangian (with multiplier $\Lambda \geq 0$):

$$\mathcal{L} = w^y L^y + w^o L^o + \Lambda \left(\bar{E}^\rho - (\theta^y L^y)^\rho - (\theta^o L^o)^\rho \right).$$

Assuming an interior solution (which holds for strictly positive wages and shifters), the FOCs are:

$$\frac{\partial \mathcal{L}}{\partial L^y} = 0 \Longrightarrow w^y = \Lambda \, \rho \, (\theta^y)^\rho \, (L^y)^{\rho - 1}, \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial L^o} = 0 \Longrightarrow w^o = \Lambda \, \rho \, (\theta^o)^\rho \, (L^o)^{\rho - 1}. \tag{21}$$

Dividing (21) by (20) yields

$$\frac{w^o}{w^y} = \left(\frac{\theta^o}{\theta^y} \right)^\rho \left(\frac{L^o}{L^y} \right)^{\rho - 1}. \tag{22}$$

Step 3: Derive conditional relative factor demands: Note that

$$\rho - 1 = \frac{\eta - 1}{\eta} - 1 = -\frac{1}{\eta}.$$

Substituting into Equation (22) gives

$$\frac{w^o}{w^y} = \left(\frac{\theta^o}{\theta^y}\right)^{\frac{\eta-1}{\eta}} \left(\frac{L^o}{L^y}\right)^{-\frac{1}{\eta}} \quad \Longrightarrow \quad \left(\frac{L^o}{L^y}\right)^{\frac{1}{\eta}} = \left(\frac{\theta^o}{\theta^y}\right)^{\frac{\eta-1}{\eta}} \left(\frac{w^y}{w^o}\right).$$

Raising both sides to the power η yields the within-sector conditional relative demand:

$$\frac{L^o}{L^y} = \left(\frac{\theta^o}{\theta^y}\right)^{\eta-1} \left(\frac{w^y}{w^o}\right)^{\eta},$$

which is Equation (16). This proves part (iii).

Step 4: Solve for the unit cost of one unit of E (set $\bar{E} = 1$): Because Equation (13) is homogeneous of degree one, the unit cost of producing $\bar{E}$ is $\bar{E}$ times the unit cost of producing one unit. Thus, it suffices to set $\bar{E} = 1$ and compute the minimum cost.

Let $\bar{E} = 1$, so the binding constraint is:

$$(\theta^y L^y)^\rho + (\theta^o L^o)^\rho = 1. \tag{23}$$

Define the cost share of each input at the optimum:

$$s^y \equiv \frac{w^y L^y}{w^y L^y + w^o L^o}, \qquad s^o \equiv \frac{w^o L^o}{w^y L^y + w^o L^o}, \qquad s^y + s^o = 1.$$

Multiply (20) by L^y and (21) by L^o to get

$$w^y L^y = \Lambda \rho (\theta^y L^y)^\rho, \tag{24}$$

$$w^o L^o = \Lambda \rho (\theta^o L^o)^\rho. \tag{25}$$

Adding (24) and (25) and using the unit constraint (23) yields:

$$w^y L^y + w^o L^o = \Lambda\rho\big[(\theta^y L^y)^\rho + (\theta^o L^o)^\rho\big] = \Lambda\rho. \tag{26}$$

Therefore the cost shares equal

$$s^y = \frac{w^y L^y}{w^y L^y + w^o L^o} = \frac{(\theta^y L^y)^\rho}{(\theta^y L^y)^\rho + (\theta^o L^o)^\rho} = (\theta^y L^y)^\rho, \tag{27}$$

and similarly $s^o = (\theta^o L^o)^\rho$. Using Equation (23), indeed $s^y + s^o = 1$.

Next, use Equation (24) together with Equation (26):

$$w^y L^y = \Lambda\rho(\theta^y L^y)^\rho = (\Lambda\rho)\, s^y.$$

Since $\Lambda\rho = w^y L^y + w^o L^o$ by Equation (26), this identity is tautological; however it allows us to express L^y and L^o in terms of shares:

$$(\theta^y L^y)^\rho = s^y \quad \Longrightarrow \quad L^y = \frac{(s^y)^{1/\rho}}{\theta^y}, \qquad L^o = \frac{(s^o)^{1/\rho}}{\theta^o}.$$

Substituting into the unit cost (minimum expenditure) gives

$$\omega \equiv \min\{w^y L^y + w^o L^o : (23)\} = w^y \frac{(s^y)^{1/\rho}}{\theta^y} + w^o \frac{(s^o)^{1/\rho}}{\theta^o}, \qquad s^o = 1 - s^y. \tag{28}$$

Now minimise (28) over $s^y \in (0,1)$. The first-order condition is:

$$\frac{\partial \omega}{\partial s^y} = w^y \frac{1}{\theta^y}\frac{1}{\rho}(s^y)^{\frac{1}{\rho}-1} - w^o \frac{1}{\theta^o}\frac{1}{\rho}(1-s^y)^{\frac{1}{\rho}-1} = 0,$$

so

$$\frac{s^y}{1-s^y} = \left(\frac{w^y/\theta^y}{w^o/\theta^o}\right)^{\rho/(\rho-1)}. \tag{29}$$

Recall $\rho - 1 = -1/\eta$, so $\rho/(\rho - 1) = -(\eta - 1)$. Therefore

$$\frac{s^y}{s^o} = \frac{s^y}{1 - s^y} = \left(\frac{w^y/\theta^y}{w^o/\theta^o}\right)^{1-\eta}. \tag{30}$$

Solving Equation (30) for shares gives:

$$s^y = \frac{(w^y/\theta^y)^{1-\eta}}{(w^y/\theta^y)^{1-\eta} + (w^o/\theta^o)^{1-\eta}}, \qquad s^o = \frac{(w^o/\theta^o)^{1-\eta}}{(w^y/\theta^y)^{1-\eta} + (w^o/\theta^o)^{1-\eta}}. \tag{31}$$

Finally, plug (31) into (28). Using $1/\rho = \eta/(\eta - 1)$ and $s^{1/\rho} = s^{\eta/(\eta-1)}$, one obtains after straightforward algebra the standard CES dual form:

$$\omega = \left[\left(\frac{w^y}{\theta^y}\right)^{1-\eta} + \left(\frac{w^o}{\theta^o}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{32}$$

which is exactly Equation (14). This proves part (i).

Step 5: Unit cost of Y and pricing under perfect competition: Since $Y = AE$ with $A > 0$, producing one unit of Y requires $1/A$ units of E. Hence the unit cost of Y is:

$$c = \frac{1}{A}\omega.$$

Under perfect competition (zero profits), the output price equals unit cost, so $P = c$. Substituting ω from (32) yields Equation (15). This proves part (ii).

Collecting Steps 3–5 completes the proof. ■

Lemma 1 follows from standard CES duality. In particular, the conditional relative demand (17) implies that, within sector j, the employment of older workers relative to younger workers increases when: i) older-worker effective productivity $\theta_j^o(D, \lambda)$ rises, and/or ii) the relative cost of older labour falls, i.e. w_j^o/w_j^y declines. The elasticity parameter η governs the strength of these substitutions: a larger η implies that sectoral age composition responds more sharply to changes in relative efficiencies and relative wages.

Define the wage bill shares of young and old labour in sector j as $e_j^y \equiv \frac{w_j^y L_j^y}{P_j Y_j}$ and $e_j^o \equiv \frac{w_j^o L_j^o}{P_j Y_j}$. Using CES properties,

$$e_j^y = \frac{(\theta_j^y L_j^y)^{\frac{\eta-1}{\eta}}}{(\theta_j^y L_j^y)^{\frac{\eta-1}{\eta}} + (\theta_j^o(D,\lambda) L_j^o)^{\frac{\eta-1}{\eta}}}, \tag{33}$$

$$e_j^o = \frac{(\theta_j^o(D,\lambda) L_j^o)^{\frac{\eta-1}{\eta}}}{(\theta_j^y L_j^y)^{\frac{\eta-1}{\eta}} + (\theta_j^o(D,\lambda) L_j^o)^{\frac{\eta-1}{\eta}}}. \tag{34}$$

These expressions indicates the logic that shares move with relative effective inputs.

2.2 Market clearing and equilibrium

Labour markets clear:

$$L_1^y + L_2^y = L^y, \qquad L_1^o + L_2^o = L^o. \tag{35}$$

Goods markets clear: intermediate outputs equal the final-good firm's demand and Q equals final production.

Definition 1 (Competitive equilibrium) *Given* (L^y, L^o) *and digital adoption* $D \geq 1$*, a competitive equilibrium is a collection*

$$\{Y_j, E_j, L_j^y, L_j^o, Q\}_{j=1,2} \text{ and } \{P_j, w_j^y, w_j^o\}_{j=1,2}$$

such that:

1. *Given* (P_1, P_2)*, the final-good producer minimises cost subject to Equation* (3)*, implying Equation* (4).
2. *Given* (P_j, w_j^y, w_j^o)*, each intermediate producer minimises costs subject to Equation* (13)*, implying Equations* (**??**) *and* (**??**).

3. *Age-specific labour allocations satisfy market clearing* (35).

4. *Age-specific sectoral shares* (x^y, x^o) *satisfy the sector-choice condition* (9) *with cost wedges given by Equation* (11)–(12).

5. *Goods markets clear and the numeraire price is one.*

2.3 A reduced-form equilibrium restriction: age-specific sorting

We now derive the "sorting" equation, but now the wedges arise from the microfounded choice block and depend on D and λ.

Let x^y and x^o be defined in (8). From Equation (10), wage ratios satisfy

$$\ln\left(\frac{w_1^y}{w_2^y}\right) = \overline{\Delta\tau}^y + \nu_y \ln\left(\frac{x^y}{1-x^y}\right), \tag{36}$$

$$\ln\left(\frac{w_1^o}{w_2^o}\right) = \overline{\Delta\tau}^o - \kappa \ln D + \chi\lambda + \nu_o \ln\left(\frac{x^o}{1-x^o}\right). \tag{37}$$

On the other hand, within-sector wage ratios are pinned down by technology and within-sector allocations. From the CES first-order conditions (equivalently from Lemma 1),

$$\frac{w_j^y}{w_j^o} = \left(\frac{\theta_j^y}{\theta_j^o(D,\lambda)}\right)^{\frac{\eta-1}{\eta}} \left(\frac{L_j^o}{L_j^y}\right)^{\frac{1}{\eta}}. \tag{38}$$

Taking the ratio of (38) across sectors and expressing (L_j^o/L_j^y) in terms of (x^o, x^y) yields a compact restriction.

Lemma 2 (Sorting restriction with logit mobility) *Let* $x^y, x^o \in (0,1)$ *be sector-1 employment shares for young and old workers, and define the log-odds*

$$\Omega^y \equiv \ln\left(\frac{x^y}{1-x^y}\right), \qquad \Omega^o \equiv \ln\left(\frac{x^o}{1-x^o}\right).$$

Suppose sectoral production is CES in age-specific labour with elasticity $\eta > 0$*,* $\eta \neq 1$*, and sector choice obeys the logit odds condition*

$$\Omega^a = \frac{1}{\nu_a}\left[\ln\!\left(\frac{w_1^a}{w_2^a}\right) - \Delta\tau^a(D,\lambda)\right], \qquad a \in \{y,o\}, \tag{39}$$

where $\nu_a > 0$ *and* $\Delta\tau^a \equiv \tau_1^a - \tau_2^a$*. Then any competitive equilibrium satisfies*

$$\big(1+\eta\nu_o\big)\,\Omega^o - \big(1+\eta\nu_y\big)\,\Omega^y = (\eta-1)\ln\!\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right) + \eta\Big(\Delta\tau^y(D,\lambda) - \Delta\tau^o(D,\lambda)\Big). \tag{40}$$

In the maintained parameterisation $\Delta\tau^y(D,\lambda) = \overline{\Delta\tau}^y$ *and* $\Delta\tau^o(D,\lambda) = \overline{\Delta\tau}^o - \kappa\ln D + \chi\lambda$*, Equation* (40) *becomes*

$$\big(1+\eta\nu_o\big)\,\Omega^o - \big(1+\eta\nu_y\big)\,\Omega^y = (\eta-1)\ln\!\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right) + \eta\Big(\overline{\Delta\tau}^y - \overline{\Delta\tau}^o + \kappa\ln D - \chi\lambda\Big). \tag{41}$$

Proof. *Step 1: CES within-sector conditional relative demand*: From the CES production block, cost minimisation (or profit maximisation) implies the standard within-sector conditional relative demand (which is exactly Lemma **??**):

$$\frac{L_j^o}{L_j^y} = \left(\frac{\theta_j^o(D,\lambda)}{\theta_j^y}\right)^{\eta-1}\left(\frac{w_j^y}{w_j^o}\right)^{\eta}, \qquad j \in \{1,2\}. \tag{42}$$

Step 2: Take the ratio across sectors and rewrite in terms of employment shares: Take Equation (42) for $j = 1$ and divide it by the same expression for $j = 2$:

$$\frac{(L_1^o/L_1^y)}{(L_2^o/L_2^y)} = \left[\frac{\theta_1^o(D,\lambda)/\theta_1^y}{\theta_2^o(D,\lambda)/\theta_2^y}\right]^{\eta-1}\left[\frac{(w_1^y/w_1^o)}{(w_2^y/w_2^o)}\right]^{\eta}. \tag{43}$$

Now use market clearing for each age group:

$$L_1^y = x^y L^y, \quad L_2^y = (1-x^y)L^y, \qquad L_1^o = x^o L^o, \quad L_2^o = (1-x^o)L^o.$$

Hence

$$\frac{L_1^o/L_1^y}{L_2^o/L_2^y} = \frac{\frac{x^o L^o}{x^y L^y}}{\frac{(1-x^o)L^o}{(1-x^y)L^y}} = \frac{x^o}{1-x^o} \cdot \frac{1-x^y}{x^y} = \frac{\frac{x^o}{1-x^o}}{\frac{x^y}{1-x^y}} = \exp(\Omega^o - \Omega^y).$$

Also note

$$\frac{(w_1^y/w_1^o)}{(w_2^y/w_2^o)} = \frac{w_1^y/w_2^y}{w_1^o/w_2^o}.$$

Substitute both identities into Equation (43) and take logs:

$$\begin{aligned}\Omega^o - \Omega^y &= (\eta - 1)\ln\left[\frac{\theta_1^o(D,\lambda)/\theta_1^y}{\theta_2^o(D,\lambda)/\theta_2^y}\right] + \eta\left[\ln\left(\frac{w_1^y}{w_2^y}\right) - \ln\left(\frac{w_1^o}{w_2^o}\right)\right] \\ &= (\eta - 1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right) + \eta\left[\ln\left(\frac{w_1^y}{w_2^y}\right) - \ln\left(\frac{w_1^o}{w_2^o}\right)\right]. \end{aligned} \tag{44}$$

Step 3: Substitute the logit sector-choice (mobility) relations: From the logit odds condition (39), rearrange to express relative wages as

$$\ln\left(\frac{w_1^a}{w_2^a}\right) = \Delta\tau^a(D,\lambda) + \nu_a\,\Omega^a, \qquad a \in \{y, o\}. \tag{45}$$

Plug Equation (45) into Equation (44):

$$\Omega^o - \Omega^y = (\eta - 1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right) + \eta\Big[\Delta\tau^y(D,\lambda) + \nu_y\Omega^y - \Delta\tau^o(D,\lambda) - \nu_o\Omega^o\Big]. \tag{46}$$

Step 4: Collect terms in Ω^o and Ω^y: Move the $\eta(\nu_y\Omega^y - \nu_o\Omega^o)$ term to the left side:

$$\Omega^o - \Omega^y - \eta\nu_y\Omega^y + \eta\nu_o\Omega^o = (\eta - 1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right) + \eta\Big(\Delta\tau^y(D,\lambda) - \Delta\tau^o(D,\lambda)\Big).$$

Combine coefficients:

$$(1+\eta\nu_o)\Omega^o-(1+\eta\nu_y)\Omega^y = (\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)+\eta\Big(\Delta\tau^y(D,\lambda)-\Delta\tau^o(D,\lambda)\Big),$$

which is exactly Equation (40). Substituting the maintained forms for $\Delta\tau^y$ and $\Delta\tau^o$ immediately gives Equation (41). ■

Lemma 2 provides the key mapping from primitives to age-specific structural change. It expresses a weighted difference of the age-specific log-odds of sector-1 employment as a transparent function of (i) relative age-augmenting technologies across sectors and (ii) age-specific mobility wedges. The lemma nests the benchmark formulation as a special case. In particular, if one shuts down the micro-founded dispersion by imposing a common logit scale (e.g., $\nu_y = \nu_o \equiv \nu$) and interprets $\Delta\tau^y$ and $\Delta\tau^o$ as reduced-form wedges, then Equation (40) collapses to a simple odds-difference restriction of the benchmark type (up to the scaling factor $1+\eta\nu$). Setting $\nu = 1$ yields the closest one-to-one analogue.

Corollary 1 (Simple odds-difference form under common dispersion) *If $\nu_y = \nu_o \equiv \nu$, then the weighted sorting restriction* (40) *implies*

$$\Omega^o-\Omega^y = \frac{1}{1+\eta\nu}\left\{(\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)+\eta\Big(\Delta\tau^y(D,\lambda)-\Delta\tau^o(D,\lambda)\Big)\right\}. \tag{47}$$

Proof. *Step 1: Start from the weighted restriction*: Lemma 2 yields the equilibrium restriction

$$(1+\eta\nu_o)\Omega^o-(1+\eta\nu_y)\Omega^y = (\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)+\eta\Big(\Delta\tau^y(D,\lambda)-\Delta\tau^o(D,\lambda)\Big), \tag{48}$$

where $\Omega^a \equiv \ln\left(\frac{x^a}{1-x^a}\right)$ for $a \in \{y, o\}$.

Step 2: Impose the common-dispersion condition: Assume $\nu_y = \nu_o \equiv \nu$. Then the left-hand side of Equation (48) becomes:

$$\begin{aligned}(1+\eta\nu_o)\Omega^o-(1+\eta\nu_y)\Omega^y &= (1+\eta\nu)\Omega^o-(1+\eta\nu)\Omega^y\\ &= (1+\eta\nu)\big(\Omega^o-\Omega^y\big).\end{aligned} \tag{49}$$

Step 3: Divide both sides by $(1+\eta\nu)$: Substituting Equation (49) into Equation (48) gives

$$(1+\eta\nu)\big(\Omega^o-\Omega^y\big)=(\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)+\eta\Big(\Delta\tau^y(D,\lambda)-\Delta\tau^o(D,\lambda)\Big).$$

Since $1+\eta\nu>0$, divide both sides by $(1+\eta\nu)$ to obtain

$$\Omega^o-\Omega^y=\frac{1}{1+\eta\nu}\left\{(\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)+\eta\Big(\Delta\tau^y(D,\lambda)-\Delta\tau^o(D,\lambda)\Big)\right\},$$

which is exactly Equation (47). ■

Proposition 1 (Digital adoption reduces age segmentation) *Assume the common-dispersion condition* $\nu_y=\nu_o\equiv\nu>0$*, so that*

$$\Omega^a\equiv\ln\left(\frac{x^a}{1-x^a}\right),\qquad a\in\{y,o\}.$$

Holding (L^y,L^o) *fixed, an increase in* D *raises the relative odds that old workers are employed in sector 1 (compared with young workers), i.e. increases* $\Omega^o-\Omega^y$*, if and only if*

$$\eta\kappa+(\eta-1)(\mu_1-\mu_2)\;>\;0. \tag{50}$$

Equivalently,

$$\frac{\partial}{\partial\ln D}\Big(\Omega^o-\Omega^y\Big)=\frac{\eta\kappa+(\eta-1)(\mu_1-\mu_2)}{1+\eta\nu}.$$

In particular, if $\kappa > 0$ *and* $\mu_1 \geq \mu_2$*, then* (50) *holds.*

Proof. *Step 1: Start from the odds-difference restriction*: Under $\nu_y = \nu_o \equiv \nu$,

$$(1+\eta\nu)\big(\Omega^o - \Omega^y\big) = (\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right) + \eta\Big(\Delta\tau^y(D,\lambda) - \Delta\tau^o(D,\lambda)\Big). \tag{51}$$

Step 2: Impose the maintained functional forms for mobility wedges: By assumption,

$$\Delta\tau^y(D,\lambda) = \overline{\Delta\tau}^y, \qquad \Delta\tau^o(D,\lambda) = \overline{\Delta\tau}^o - \kappa\ln D + \chi\lambda.$$

Hence

$$\Delta\tau^y(D,\lambda) - \Delta\tau^o(D,\lambda) = \big(\overline{\Delta\tau}^y - \overline{\Delta\tau}^o - \chi\lambda\big) + \kappa\ln D. \tag{52}$$

Therefore, the wedge component on the right-hand side of (51) contributes

$$\eta\Big(\Delta\tau^y - \Delta\tau^o\Big) = \eta\big(\overline{\Delta\tau}^y - \overline{\Delta\tau}^o - \chi\lambda\big) + \eta\kappa\ln D.$$

Step 3: Impose the maintained functional forms for old-worker efficiency: Recall

$$\theta_j^o(D,\lambda) = \bar{\theta}_j^o(1-\lambda)^{\rho_j}D^{\mu_j}, \qquad \mu_j \geq 0.$$

Then

$$\frac{\theta_1^o(D,\lambda)}{\theta_2^o(D,\lambda)} = \frac{\bar{\theta}_1^o}{\bar{\theta}_2^o}\,(1-\lambda)^{\rho_1-\rho_2}\,D^{\mu_1-\mu_2}.$$

Hence

$$\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right) = \underbrace{\ln\left(\frac{\bar{\theta}_1^o\,\theta_2^y}{\theta_1^y\,\bar{\theta}_2^o}\right) + (\rho_1-\rho_2)\ln(1-\lambda)}_{\text{independent of } D} + (\mu_1-\mu_2)\ln D. \tag{53}$$

Therefore, the technology component on the right-hand side of Equation

(51) contributes

$$(\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)=\text{constant in } D+(\eta-1)(\mu_1-\mu_2)\ln D.$$

Step 4: Differentiate with respect to $\ln D$: Differentiate both sides of Equation (51) w.r.t. $\ln D$. The left-hand side gives:

$$\frac{\partial}{\partial \ln D}\Big[(1+\eta\nu)(\Omega^o-\Omega^y)\Big]=(1+\eta\nu)\frac{\partial}{\partial \ln D}(\Omega^o-\Omega^y),$$

since ν is constant. The right-hand side derivative is obtained from Equations (52) and (53):

$$\frac{\partial}{\partial \ln D}\left[(\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)+\eta(\Delta\tau^y-\Delta\tau^o)\right]=(\eta-1)(\mu_1-\mu_2)+\eta\kappa.$$

Thus

$$(1+\eta\nu)\frac{\partial}{\partial \ln D}(\Omega^o-\Omega^y)=\eta\kappa+(\eta-1)(\mu_1-\mu_2), \tag{54}$$

or equivalently

$$\frac{\partial}{\partial \ln D}(\Omega^o-\Omega^y)=\frac{\eta\kappa+(\eta-1)(\mu_1-\mu_2)}{1+\eta\nu}. \tag{55}$$

Step 5: Sign condition ("if and only if"): Because $1+\eta\nu>0$, the sign of Equation (55) is determined entirely by the numerator. Therefore,

$$\frac{\partial}{\partial \ln D}(\Omega^o-\Omega^y)>0 \quad\Longleftrightarrow\quad \eta\kappa+(\eta-1)(\mu_1-\mu_2)>0,$$

which is exactly (50). Finally, if $\kappa>0$ and $\mu_1\geq\mu_2$, then the numerator is strictly positive, so adoption necessarily increases $\Omega^o-\Omega^y$.

This completes the proof. ■

Proposition 1 is intuitive. Digital adoption affects age-specific sectoral allocation through two distinct mechanisms. First, it reduces the old-specific mobility barrier, captured by $\kappa > 0$, thereby lowering the relative wage compensation required to attract older workers into sector 1. Second, it changes the sectoral pattern of older-worker efficiency via the differential complementarity term $\mu_1 - \mu_2$, which shifts the relative effective productivity of older labour toward the sector in which digital tools are more valuable. When $\eta\kappa + (\eta - 1)(\mu_1 - \mu_2) > 0$, these forces jointly increase the difference in log-odds $\Omega^o - \Omega^y$, implying a reallocation of older workers toward sector 1 relative to younger workers and, in this sense, a reduction in age segmentation.

Ageing enters the sorting restriction in two distinct ways. First, it directly raises the old-specific mobility barrier through the term $\chi\lambda$ embedded in $\Delta\tau^o(D, \lambda)$, reflecting that sector switching or job search may become more costly as the workforce becomes older. Second, it affects the relative efficiency of older workers through the "effective ageing" component $(1-\lambda)^{\rho_j}$ in $\theta_j^o(D, \lambda)$, which allows ageing to erode older-worker productivity in a sector-dependent manner. Differentiating the (weighted) sorting restriction implied by Lemma 2 with respect to λ therefore delivers a transparent prediction for how demographic shifts reshape age-specific sectoral allocation.

Proposition 2 (Ageing and sorting) *Assume the common-dispersion condition $\nu_y = \nu_o \equiv \nu > 0$ and fix D. Let $\Omega^a \equiv \ln\left(\frac{x^a}{1-x^a}\right)$ for $a \in \{y, o\}$. Under $\Delta\tau^y(D, \lambda) = \overline{\Delta\tau}^y$ and $\Delta\tau^o(D, \lambda) = \overline{\Delta\tau}^o - \kappa \ln D + \chi\lambda$, and $\theta_j^o(D, \lambda) = \bar{\theta}_j^o(1-\lambda)^{\rho_j} D^{\mu_j}$, an increase in the ageing index λ raises the relative odds that old workers are employed in sector 1 (relative to young workers), i.e. increases $\Omega^o - \Omega^y$, if and only if*

$$\frac{\eta - 1}{1 - \lambda}(\rho_2 - \rho_1) > \eta\,\chi. \tag{56}$$

Equivalently,

$$\frac{\partial}{\partial\lambda}\left(\Omega^o - \Omega^y\right) = \frac{1}{1 + \eta\nu}\left[\frac{\eta - 1}{1 - \lambda}(\rho_2 - \rho_1) - \eta\chi\right], \tag{57}$$

so the sign is governed by the bracketed term.

In particular, if ageing worsens old-worker effectiveness more strongly in sector 2 than sector 1 (i.e. $\rho_2 > \rho_1$) and the mobility-barrier channel is weak (small χ), then ageing reallocates older workers away from sector 2 and toward sector 1, raising $\Omega^o - \Omega^y$.

Proof. *Step 1: Start from the simple odds-difference restriction*: Under $\nu_y = \nu_o \equiv \nu$,

$$(1+\eta\nu)\big(\Omega^o-\Omega^y\big) = (\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)+\eta\Big(\Delta\tau^y(D,\lambda)-\Delta\tau^o(D,\lambda)\Big). \quad (58)$$

We treat D as fixed and differentiate with respect to λ.

Step 2: Differentiate the mobility-wedge component: By assumption,

$$\Delta\tau^y(D,\lambda) = \overline{\Delta\tau}^y \quad \Rightarrow \quad \frac{\partial}{\partial\lambda}\Delta\tau^y(D,\lambda) = 0,$$

and

$$\Delta\tau^o(D,\lambda) = \overline{\Delta\tau}^o - \kappa\ln D + \chi\lambda \quad \Rightarrow \quad \frac{\partial}{\partial\lambda}\Delta\tau^o(D,\lambda) = \chi.$$

Therefore,

$$\frac{\partial}{\partial\lambda}\Big(\Delta\tau^y - \Delta\tau^o\Big) = -\chi, \qquad \Rightarrow \qquad \frac{\partial}{\partial\lambda}\Big[\eta(\Delta\tau^y - \Delta\tau^o)\Big] = -\eta\chi. \quad (59)$$

Step 3: Differentiate the technology component: Only the old-efficiency terms depend on λ. Using $\theta_j^o(D,\lambda) = \bar{\theta}_j^o(1-\lambda)^{\rho_j}D^{\mu_j}$ and holding D fixed,

$$\ln\theta_j^o(D,\lambda) = \ln\bar{\theta}_j^o + \rho_j\ln(1-\lambda) + \mu_j\ln D,$$

so

$$\frac{\partial}{\partial\lambda}\ln\theta_j^o(D,\lambda) = \rho_j\frac{\partial}{\partial\lambda}\ln(1-\lambda) = \rho_j\left(-\frac{1}{1-\lambda}\right) = -\frac{\rho_j}{1-\lambda}. \quad (60)$$

Since θ_1^y and θ_2^y do not depend on λ, we have:

$$\frac{\partial}{\partial\lambda}\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right) = \frac{\partial}{\partial\lambda}\Big[\ln\theta_1^o(D,\lambda) - \ln\theta_2^o(D,\lambda)\Big]$$
$$= \left(-\frac{\rho_1}{1-\lambda}\right) - \left(-\frac{\rho_2}{1-\lambda}\right) = \frac{\rho_2-\rho_1}{1-\lambda}. \tag{61}$$

Multiplying by $(\eta-1)$ yields:

$$\frac{\partial}{\partial\lambda}\left[(\eta-1)\ln\left(\frac{\theta_1^o(D,\lambda)\,\theta_2^y}{\theta_1^y\,\theta_2^o(D,\lambda)}\right)\right] = \frac{\eta-1}{1-\lambda}(\rho_2-\rho_1). \tag{62}$$

Step 4: Differentiate the full restriction and solve for $\partial_\lambda(\Omega^o-\Omega^y)$: Differentiate Equation (58) w.r.t. λ. The left-hand side gives:

$$\frac{\partial}{\partial\lambda}\Big[(1+\eta\nu)(\Omega^o-\Omega^y)\Big] = (1+\eta\nu)\frac{\partial}{\partial\lambda}(\Omega^o-\Omega^y),$$

since $1+\eta\nu$ is constant. The right-hand side derivative is the sum of Equations (62) and (59):

$$\frac{\partial}{\partial\lambda}\{\text{RHS}\} = \frac{\eta-1}{1-\lambda}(\rho_2-\rho_1) - \eta\chi.$$

Therefore,

$$(1+\eta\nu)\frac{\partial}{\partial\lambda}(\Omega^o-\Omega^y) = \frac{\eta-1}{1-\lambda}(\rho_2-\rho_1) - \eta\chi,$$

which implies Equation (57).

Step 5: Sign condition (“if and only if”): Because $1+\eta\nu>0$, we have:

$$\frac{\partial}{\partial\lambda}(\Omega^o-\Omega^y) > 0 \quad\Longleftrightarrow\quad \frac{\eta-1}{1-\lambda}(\rho_2-\rho_1)-\eta\chi > 0 \quad\Longleftrightarrow\quad \frac{\eta-1}{1-\lambda}(\rho_2-\rho_1) > \eta\chi,$$

which is exactly (56). This completes the proof. ■

Proposition 2 shows that the direction of the ageing effect is governed by a simple trade-off between a mobility channel and a technology channel.

On the one hand, ageing raises the old-specific mobility barrier through χ, which tends to reduce $\Omega^o - \Omega^y$ by limiting the reallocation of older workers into sector 1. On the other hand, ageing changes the relative effectiveness of older labour across sectors through the term $\rho_2 - \rho_1$: if ageing erodes older-worker efficiency more strongly in sector 2 than in sector 1 (i.e. $\rho_2 > \rho_1$), then the technology term contributes $\frac{\eta-1}{1-\lambda}(\rho_2 - \rho_1)$ and pushes older workers away from sector 2 and toward sector 1, thereby increasing $\Omega^o - \Omega^y$. The sign condition (56) makes this comparison explicit, illustrating why a purely reduced-form wedge can mask the underlying mechanism, whereas separating mobility frictions from age-biased technology delivers transparent predictions.

A natural implication is that adoption matters more when the economy is older.

Corollary 2 (No interaction under the baseline mobility specification) *Assume the common-dispersion condition $\nu_y = \nu_o \equiv \nu > 0$ and fix $D \geq 1$. Under $\Delta\tau^y(D,\lambda) = \overline{\Delta\tau}^y$ and $\Delta\tau^o(D,\lambda) = \overline{\Delta\tau}^o - \kappa \ln D + \chi\lambda$, and $\theta^o_j(D,\lambda) = \bar{\theta}^o_j(1-\lambda)^{\rho_j} D^{\mu_j}$, the marginal effect of D on the odds-difference $\Omega^o - \Omega^y$ satisfies*

$$\frac{\partial}{\partial \ln D}\big(\Omega^o - \Omega^y\big) = \frac{\eta\kappa + (\eta-1)(\mu_1 - \mu_2)}{1+\eta\nu}, \tag{63}$$

which is constant in λ. Consequently,

$$\frac{\partial^2}{\partial\lambda\,\partial \ln D}\big(\Omega^o - \Omega^y\big) = 0. \tag{64}$$

Proof. Under $\nu_y = \nu_o \equiv \nu$,

$$(1+\eta\nu)\big(\Omega^o - \Omega^y\big) = (\eta-1)\ln\left(\frac{\theta^o_1(D,\lambda)\theta^y_2}{\theta^y_1\theta^o_2(D,\lambda)}\right) + \eta\big(\Delta\tau^y(D,\lambda) - \Delta\tau^o(D,\lambda)\big). \tag{65}$$

Differentiate Equation (65) with respect to $\ln D$.

Technology term. Since $\theta^o_j(D,\lambda) = \bar{\theta}^o_j(1-\lambda)^{\rho_j} D^{\mu_j}$,

$$\ln \theta^o_j(D,\lambda) = \ln \bar{\theta}^o_j + \rho_j \ln(1-\lambda) + \mu_j \ln D \quad \Rightarrow \quad \frac{\partial}{\partial \ln D} \ln \theta^o_j(D,\lambda) = \mu_j,$$

hence

$$\frac{\partial}{\partial \ln D} \ln\left(\frac{\theta^o_1(D,\lambda)\theta^y_2}{\theta^y_1 \theta^o_2(D,\lambda)}\right) = \mu_1 - \mu_2. \tag{66}$$

Wedge term. With $\Delta\tau^y = \overline{\Delta\tau}^y$ and $\Delta\tau^o = \overline{\Delta\tau}^o - \kappa \ln D + \chi\lambda$,

$$\frac{\partial}{\partial \ln D}\big(\Delta\tau^y - \Delta\tau^o\big) = \frac{\partial}{\partial \ln D}\big(\overline{\Delta\tau}^y - \overline{\Delta\tau}^o + \kappa \ln D - \chi\lambda\big) = \kappa. \tag{67}$$

Combine Equations (66) and (67) and differentiate Equation (65):

$$(1+\eta\nu)\frac{\partial}{\partial \ln D}\big(\Omega^o - \Omega^y\big) = (\eta-1)(\mu_1-\mu_2) + \eta\kappa,$$

which implies Equation (63). Since the right-hand side of (63) contains no λ, the cross-partial Equation (64) follows immediately. ■

Assumption 2 (Ageing-amplified adoption in old mobility) *Replace the old mobility wedge by:*

$$\Delta\tau^o(D,\lambda) = \overline{\Delta\tau}^o - \kappa\big(1+\varphi\,\chi\lambda\big)\ln D + \chi\lambda, \qquad \kappa>0,\ \varphi>0,\ \chi\geq 0, \tag{68}$$

while keeping $\Delta\tau^y(D,\lambda) = \overline{\Delta\tau}^y$ *and* $\theta^o_j(D,\lambda)$ *unchanged.*

Corollary 3 (Stronger adoption effect under ageing) *Assume* $\nu_y = \nu_o \equiv \nu > 0$ *and fix primitives. Under Assumption 2, the marginal effect of* D *on* $\Omega^o - \Omega^y$ *is increasing in* λ *whenever* $\chi > 0$*:*

$$\frac{\partial}{\partial \ln D}\big(\Omega^o - \Omega^y\big) = \frac{\eta\kappa\big(1+\varphi\,\chi\lambda\big) + (\eta-1)(\mu_1-\mu_2)}{1+\eta\nu}, \tag{69}$$

and hence

$$\frac{\partial^2}{\partial\lambda\,\partial\ln D}\big(\Omega^o-\Omega^y\big)=\frac{\eta\kappa\varphi\chi}{1+\eta\nu}\;>\;0\qquad\textit{whenever}\;\chi>0. \tag{70}$$

Therefore, for larger λ—in particular for λ close to one—the absolute marginal effect of D on the sorting object $\Omega^o-\Omega^y$ is larger.

Proof. Start from Equation (65). The technology derivative with respect to $\ln D$ is unchanged and given by Equation (66), i.e. $(\mu_1-\mu_2)$.

Under Assumption 2,

$$\Delta\tau^y-\Delta\tau^o=\overline{\Delta\tau}^y-\overline{\Delta\tau}^o+\kappa\big(1+\varphi\chi\lambda\big)\ln D-\chi\lambda.$$

Hence,

$$\frac{\partial}{\partial\ln D}\big(\Delta\tau^y-\Delta\tau^o\big)=\kappa\big(1+\varphi\chi\lambda\big). \tag{71}$$

Differentiating Equation (65) with respect to $\ln D$ yields:

$$(1+\eta\nu)\frac{\partial}{\partial\ln D}\big(\Omega^o-\Omega^y\big)=(\eta-1)(\mu_1-\mu_2)+\eta\kappa\big(1+\varphi\chi\lambda\big),$$

which gives Equation (69). Differentiating Equation (69) with respect to λ gives (70). Since $1+\eta\nu>0$, the cross-partial is strictly positive whenever $\chi>0$, establishing that the adoption effect is larger for higher λ, and thus in particular when λ is close to one. ■

Corollary 3 formalises a simple complementarity between ageing and digital adoption. When ageing raises old-worker mobility frictions ($\chi>0$), digital adoption has more scope to relax those frictions under the ageing-amplified wedge (68). As a result, the marginal effect of D on the age-sorting object $\Omega^o-\Omega^y$ is increasing in λ: in more aged economies, adoption delivers a larger reduction in age segmentation by facilitating the reallocation of older workers across sectors. The sector-biased productivity erosion ($\rho_2>\rho_1$) provides an additional motive for reallocation, but the mechanism in Corollary 3 operates through mobility frictions—it is precisely when reallocation becomes harder with ageing that digital adoption becomes more valuable at the margin.

So far we focused on employment shares. A full model also needs to discipline prices and sectoral output. We now keep the algebra manageable by working with the relative price

$$p \equiv \frac{P_1}{P_2}. \tag{72}$$

Under perfect competition, P_j equals unit cost Equation (15). Using Equation (72),

$$p = \frac{A_2}{A_1} \cdot \frac{\left[\left(\frac{w_1^y}{\theta_1^y}\right)^{1-\eta} + \left(\frac{w_1^o}{\theta_1^o(D,\lambda)}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}}{\left[\left(\frac{w_2^y}{\theta_2^y}\right)^{1-\eta} + \left(\frac{w_2^o}{\theta_2^o(D,\lambda)}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}}. \tag{73}$$

Equation (73) links the relative price to sectoral wages and technologies. Wages, in turn, are pinned down by labour allocations and the sector-choice system (36)–(37).

From Equation (4), relative nominal expenditures satisfy

$$\frac{P_1 Y_1}{P_2 Y_2} = \frac{\gamma_1}{\gamma_2} p^{1-\varepsilon}. \tag{74}$$

Because firms make zero profits under perfect competition, $P_j Y_j$ equals the wage bill in sector j:

$$P_j Y_j = w_j^y L_j^y + w_j^o L_j^o. \tag{75}$$

Thus Equatio (74) can be rewritten as a restriction involving (x^y, x^o) and wage ratios.

Combining the key blocks yields a compact representation of equilibrium: i) *Sector-choice (young)*: Equation (36); ii) *Sector-choice (old)*: Equation (37); iii) *Sorting restriction*: Equation (40) (equivalent to combining within-sector CES with sector choice); iv) *Relative price from unit costs*: Equation (73); v) *Relative demand*: Equation (74) together with Equation (75).

An object often used in empirical work is average labour productivity,

$$\bar{A} \equiv \frac{Q}{L^y + L^o}. \tag{76}$$

Because Q is homogeneous of degree one in (Y_1, Y_2) and each Y_j is homogeneous of degree one in labour inputs, $\bar{A}$ is well-defined and responds to both: i) within-sector age composition (which affects unit costs) and ii) across-sector reallocation (which changes relative output weights).

A simple way to study the sign of $\partial \bar{A}/\partial D$ and $\partial \bar{A}/\partial \lambda$ is to use the zero-profit condition (75) and the fact that the final good is the numeraire:

$$Q = \sum_{j=1}^{2} P_j Y_j = \sum_{j=1}^{2} \left(w_j^y L_j^y + w_j^o L_j^o \right). \tag{77}$$

Thus aggregate output equals aggregate labour income, and changes in productivity can be traced through wages and reallocations. Digital adoption raises $\theta_j^o(D, \lambda)$ and relaxes $\Delta\tau^o(D, \lambda)$; both effects tend to increase equilibrium old wages relative to young wages, and make those wages closer to the frictionless allocation. That combination is the sense in which adoption can hedge the productivity consequences of ageing.

To expand the model beyond a purely comparative-static exercise, we endogenises D through a simple resource allocation problem. Consider a planner who takes the decentralised equilibrium mapping $Q = Q(D, \lambda)$ as given (for fixed primitives and competitive behaviour) and chooses D to maximise welfare:

$$\max_{D \geq 1} \Big\{ Q(D, \lambda) - \Phi(D) \Big\}. \tag{78}$$

The first-order condition is

$$\frac{\partial Q(D,\lambda)}{\partial D} = \Phi'(D). \tag{79}$$

Although $Q(D,\lambda)$ is an equilibrium object, Proposition 1 already tells us the sign of the key allocation response: higher D reduces age segmentation when (50) holds. In addition, $\partial Q/\partial D$ is positive whenever adoption raises old-worker efficiency sufficiently (some $\mu_j > 0$) and/or reduces misallocation from mobility frictions ($\kappa > 0$). Thus the planner trades off a standard convex cost Φ against two production-side gains.

A useful comparative-static prediction is that the optimal D rises with ageing.

Proposition 3 (Optimal adoption rises with ageing) *Assume Assumption 1. Let $Q(D,\lambda)$ be twice continuously differentiable in a neighbourhood of an interior optimum and suppose*

$$\frac{\partial^2 Q}{\partial D\,\partial\lambda}(D,\lambda) > 0 \quad \textit{in a neighbourhood of } (D^\star(\lambda),\lambda).$$

Let $D^\star(\lambda)$ solve the planner's first-order condition

$$\frac{\partial Q(D,\lambda)}{\partial D} = \Phi'(D). \tag{80}$$

Then

$$\frac{dD^\star(\lambda)}{d\lambda} > 0. \tag{81}$$

Proof. *Step 1: Write the planner problem and the first-order condition*: The planner chooses $D \geq 1$ to maximise:

$$W(D,\lambda) \equiv Q(D,\lambda) - \Phi(D),$$

where λ is a parameter. Under Assumption 1 and interiority of the solution, a necessary condition for $D^\star(\lambda)$ is:

$$\frac{\partial W(D,\lambda)}{\partial D}\Big|_{D=D^\star(\lambda)} = 0 \quad \Longleftrightarrow \quad \frac{\partial Q(D,\lambda)}{\partial D}\Big|_{D=D^\star(\lambda)} = \Phi'\big(D^\star(\lambda)\big),$$

which is Equation (80).

Step 2: Apply the Implicit Function Theorem: Define the function:

$$F(D,\lambda) \equiv \frac{\partial Q(D,\lambda)}{\partial D} - \Phi'(D). \tag{82}$$

The first-order condition is $F(D^\star(\lambda),\lambda) = 0$.

Differentiate $F(D^\star(\lambda),\lambda) = 0$ with respect to λ:

$$\frac{\partial F}{\partial D}(D^\star(\lambda),\lambda) \cdot \frac{dD^\star(\lambda)}{d\lambda} + \frac{\partial F}{\partial \lambda}(D^\star(\lambda),\lambda) = 0.$$

Solving for $dD^\star/d\lambda$ gives:

$$\frac{dD^\star(\lambda)}{d\lambda} = -\frac{F_\lambda(D^\star(\lambda),\lambda)}{F_D(D^\star(\lambda),\lambda)}. \tag{83}$$

Compute the partial derivatives of F from Equation (82):

$$F_\lambda(D,\lambda) = \frac{\partial}{\partial \lambda}\left(\frac{\partial Q(D,\lambda)}{\partial D}\right) = \frac{\partial^2 Q(D,\lambda)}{\partial \lambda\, \partial D}, \tag{84}$$

$$F_D(D,\lambda) = \frac{\partial}{\partial D}\left(\frac{\partial Q(D,\lambda)}{\partial D} - \Phi'(D)\right) = \frac{\partial^2 Q(D,\lambda)}{\partial D^2} - \Phi''(D). \tag{85}$$

Substitute Equation (84)–(85) into Equation (83) to obtain

$$\frac{dD^\star(\lambda)}{d\lambda} = -\frac{\dfrac{\partial^2 Q(D^\star(\lambda),\lambda)}{\partial \lambda\, \partial D}}{\dfrac{\partial^2 Q(D^\star(\lambda),\lambda)}{\partial D^2} - \Phi''(D^\star(\lambda))}. \tag{86}$$

Step 3: Sign of the denominator (second-order condition): For an interior maximum, the second-order condition requires

$$\frac{\partial^2 W(D,\lambda)}{\partial D^2}\Big|_{D=D^\star(\lambda)} = \frac{\partial^2 Q(D,\lambda)}{\partial D^2}\Big|_{D=D^\star(\lambda)} - \Phi''\big(D^\star(\lambda)\big) < 0. \tag{87}$$

Thus the denominator in Equation (86) is strictly negative at the optimum.
Step 4: Sign of the numerator and conclusion: By assumption,

$$\frac{\partial^2 Q}{\partial D\,\partial\lambda}(D^\star(\lambda),\lambda) > 0,$$

so the numerator in Equation (86) is strictly positive. Combining this with the negative denominator from Equation (87) yields:

$$\frac{dD^\star(\lambda)}{d\lambda} = -\frac{(+)}{(-)} > 0,$$

which proves (81). ■

3. Conclusion

This paper studies a simple but timely question: when the workforce becomes older, can digital adoption reduce age segmentation across sectors, or does ageing deepen it? We address this question in a two-sector environment with two age groups (young and old), where (i) sectoral production uses a CES composite of young and old labour, and (ii) sectoral reallocation is imperfect because workers face age-specific mobility barriers generated by a logit sector-choice structure. Digital adoption enters in two places that matter for older workers: it can raise their sector-specific efficiency, and it can reduce their sector-switching frictions.

The model delivers a tractable "sorting" restriction that links age-specific sector-1 employment shares (in log-odds) to two sets of primitives: relative age-augmenting technologies across sectors and age-specific mobility wedges. This restriction makes the key trade-offs transparent. Digital adoption reduces age segmentation when it sufficiently relaxes old-worker mobility barriers and/or increases old-worker effective productivity more in one sector than the other. Ageing, in turn, moves sectoral allocation through two competing forces: it can make older workers harder to reallocate (a mobility channel) and it can erode their effectiveness in a sector-biased way (a technology channel). Which force dominates is pinned down by a simple inequality. Finally, when we allow the effectiveness of adoption in reducing old-worker mobility frictions to rise with ageing, the marginal impact of digital adoption on age segmentation becomes larger in more aged economies.

These results are not meant to be a full quantitative account of structural transformation in ageing societies. Rather, they provide a compact framework that clarifies why the same demographic trend can produce different labour-market outcomes across countries and industries: the answer depends on (i) how age-biased productivity differs across sectors and (ii) how mobility frictions evolve with age, and on whether digital adoption affects one or both margins.

The framework points to several policy recommendations that are practical and targeted. First, treat digital adoption as both a productivity policy and a mobility policy. Many digital policies are evaluated mainly through average productivity effects (e.g., whether firms become more efficient). Our model highlights an additional margin: adoption can also lower the effective costs older workers face when switching sectors (through search, matching, certification, and remote-work feasibility). Policies that raise adoption but ignore mobility barriers may deliver smaller distributional and reallocation gains for older workers.

Second, target adoption and re-skilling where digital complementarity is strongest. The productivity channel depends on sectoral complementarity parameters (the model's μ_j terms). This suggests that re-skilling and workplace redesign should be targeted to sectors where digital tools are most capable of raising older-worker effectiveness, rather than applied uniformly across the economy. In practice, this means prioritising training, workflow redesign, and digital support in sectors with tasks that are more modular, codifiable, or compatible with assistive technologies and remote work.

Third, reduce age-specific mobility barriers directly. If ageing raises mobility costs (captured by the χ channel), then policies that reduce switching barriers can prevent older workers from becoming trapped in shrinking sectors. Examples include: portable benefits that reduce the cost of changing employers, recognition of prior learning and credentials across industries, job-matching services designed for older workers, short-cycle training that is compatible with health constraints, and job redesign standards that reduce physical demands. The model's message is simple: when reallocation is hard, sectoral wage signals are not enough.

Fourth, expect higher returns to adoption in more aged economies. Under the interaction extension (where adoption becomes more effective at reducing frictions when those frictions are higher), the marginal reallocation benefit of adoption increases with the ageing index. This supports a life-cycle oriented approach to digital policy: economies with more pronounced ageing should place greater weight on adoption and matching infrastructure, because the same intervention can remove more misallocation when the workforce is older.

Fifth, combine adoption policies with competition and diffusion policies. If adoption is concentrated in a small set of firms, the mobility channel may not operate broadly: workers cannot move into digitally enabled jobs if such jobs are scarce. Policies that promote diffusion of digital practices—standard-setting, interoperability, support for SMEs, and training consortia—can amplify both the productivity and mobility channels.

The paper is intentionally small. It is static, abstracts from capital and from lifecycle decisions such as retirement, and represents mobility frictions in reduced form through sector-choice wedges. These simplifications are deliberate: they allow closed-form restrictions that clarify mechanisms. A natural next step is to embed the same age-specific mobility logic in a dynamic environment with capital accumulation, endogenous training, and multiple sectors, so that one can quantify transitional effects and welfare under realistic demographic paths. Another extension is to allow heterogeneity within age groups (health, wealth, education) and to discipline the mobility wedge using micro data on sector switches by age.

Even in its stripped-down form, the framework yields a clear conclusion: ageing does not mechanically imply deeper segmentation of older workers across sectors. The direction depends on whether technology erosion is sector-biased and on how mobility frictions evolve with age. Digital adoption can mitigate segmentation when it improves older-worker efficiency and, crucially, when it lowers the barriers that prevent older workers from reallocating as the economy changes.

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